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divided by 192 is 0″·005, a negligible quantity. Although, therefore, the other columns of Table VI. have not been subjected to so searching a test, I believe the results as given in Tables VII. and VIII. may be relied on to within the accidental errors due to the observations.

On the Discordant Values of the Principal Elliptic Coefficient in the Moon's Longitude. By P. H. Cowell.

The values given for the coefficient of the principal elliptic term in the Moon's longitude have varied from 22637″·15 of Hansen's theory to 22641″ 6 of Airy's tabular places. In the present note I take some of the values published during the last half-century, and I try as far as possible to trace the discordances to their source. The values in question are—

Ref. No.	Author.	Coefficient. ″	Material.	Reference.
1	Airy	22639·06	Greenwich 1750–1851	*Memoirs R.A.S.* xxix. p. 13.
2	Newcomb	·82	„ 1847–1858	Corrections to Hansen's Tables, p. 29.
3	„	·50	Greenwich and Washington 1862–1874	„ „
4	Nevill	·32	Greenwich 1862–1877	*Memoirs R.A.S.* xlviii. p. 417.
5	Cowell	·54	„ 1750–1851	*Monthly Notices*, vol. lxv. p. 147.
6	„	·46	„ 1847–1901	„ „

I shall establish the propriety of the following corrections to the above values:

Ref. No.	Solar Correction. ″	Planetary Corrections. ″	Corrected Value. ″
1	+0·16	...	22639·22
2	−0·25	+0·06	·63
3	−0·25	+0·10	·35
4	+0·05	+0·06	·43
5	...	...	·54
6	...	...	·46

It will be seen that the accordance of the six values is improved by these corrections. The range is reduced from 0″·76 to 0″·41, or, leaving out Airy's result, from 0″·50 to 0″·28.

One of my values (Ref. No. 5) is based upon the same observed places as Airy's, and the discordance is not therefore due to errors of observation. Either Airy's analysis or mine is wrong.

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Airy obtains his result by finding a correction, $-2''{\cdot}54$, to the value used in his tabular places. As Airy divides his observations into two groups only, each group covering a range of 180° in the mean anomaly, and as the coefficients of some of his terms are over $3''$ in error, an error of 10 per cent. in his correction is intelligible, and in examining the accordance of various results Airy's may therefore be left out of account. Airy, in fact, missed a great opportunity. He had material with which any short-period coefficient could certainly be obtained to within $0''{\cdot}2$ (see my result, Ref. No. 5),and for 1 per cent. additional to the labour actually expended in the reductions he might have given observed values for all coefficients several years in advance of equally good theoretical ones ; and if he had taken a year, instead of nine years, as his unit of analysis, he could not have failed to anticipate Professor Newcomb's discovery of the *Jupiter* evection term.

I will now explain the foregoing corrections. The solar corrections are intended to reduce the results to what would have been obtained if the coefficients $-8''{\cdot}44$ and $+18''{\cdot}55$ had been used for the terms $\sin(g+D)$ and $\sin(g-D)$. Both Airy and Professor Newcomb have overlooked the necessity of combining a consideration of these two terms with a discussion of the eccentricity of the Moon's orbit. Airy was fortunate in so far that the errors of the coefficients of the terms $\sin(g\pm D)$ employed by him are far smaller than those of many of his other coefficients. Hansen had published in the *Darlegung* a large correction to the coefficient of $\sin(g-D)$ before Professor Newcomb made his investigations. That the values above quoted for the coefficients in question are final may be inferred from the fact that Hansen's, Delaunay's, and Professor Brown's theories and my own observed values are in close agreement.

In *Memoirs R.A.S.* vol. xlviii. p. 315 Mr. Nevill points out that a term (in the notation of Hansen and Professor Newcomb) $\delta B \sin(g'+D)$ will produce an apparent effect $-\delta B \times 0{\cdot}70 \sin g'$. He arrives at the factor $-0{\cdot}70$ by supposing that the observations are uniformly distributed from first quarter to last quarter. I employ the factor $-0{\cdot}48$ obtained as the mean value of $\cos D$. Mr. Nevill nowhere states the point in question in more general terms than in the above special case, but I conclude that he has in fact corrected his result for the error $0''{\cdot}60 \sin(g-D)$ in the course of pp. 315–318 of his memoir. The small correction I have therefore applied to his result represents an error $0''{\cdot}10 \sin(g+D)$ only.

Coming now to the planetary corrections, results Nos. 1, 5 need no correction, as the observations extend over 100 years. In No. 6 the results extend over fifty-four years, and the planetary terms have already been applied to the individual tabular places. Therefore No. 6 needs no correction either. Also results 5 and 6 cover 150 years between them, and as they are in close agreement, the presumption is that there exists no undiscovered term capable

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of affecting the result. Planetary corrections have, therefore, only to be applied to Nos. 2, 3, 4.

(1) $+0''\cdot316 \sin (g+2\varpi-3J+7^\circ)$ calculated by M. Radau, and in *Monthly Notices*, lxv. p. 135, shown to be confirmed by the observations. The term was unknown to Professor Newcomb and to Mr. Nevill.

Ref. No.	Mean Epoch.	Value of $2\varpi-3J+7^\circ$.	θ.	Coefficient multiplied by $\frac{\sin\theta}{\theta}$.	Correction.
		°	°	″	″
2	1853·0	60	58	+0·27	−0·13
3	1868·5	270	63	+0·26	0·00
4	1870·0	255	77	+0·22	+0·06

The first line of the above table reads as follows:

Professor Newcomb's investigation (Ref. No. 2) has a mean epoch 1853·0 when the value of $2\varpi-3J+7^\circ$ takes the value 60°. During the period over which the investigation extends, $2\varpi-3J+7^\circ$ varies from $60^\circ+58^\circ$ to $60^\circ-58^\circ$. The correction required is therefore $-0''\cdot316 \frac{\sin 58^\circ}{58^\circ} \cos 60^\circ = -0''\cdot13$.

(2) $-1''\cdot1 \sin (g+2\varpi-2J)$. This is the *Jupiter* evection term discovered by Professor Newcomb, and attributed to the action of *Jupiter* by Mr. Nevill. Mr. Nevill uses $-1''\cdot4$ as the coefficient, and the correction consequently required by his result is insensible. The coefficient $-1''\cdot1$ is indicated by the observations 1750 to 1901, whereas Dr. G. W. Hill and M. Radau agree in giving $-0''\cdot9$ as the theoretical coefficient.

The term was discovered by Professor Newcomb as a wave of 17½ years from crest to crest in the values of the coefficients of sin g and cos g, as given by two investigations extending over twelve and thirteen years respectively. Under these circumstances a high degree of accuracy was of course unattainable. Professor Newcomb gives as the empirical term

$$-1''\cdot5 \sin (g+253^\circ\cdot2+21^\circ\cdot6\,(t-1868\cdot5))$$

The means of the values of h, exhibited by Professor Newcomb on p. 28 of his paper, are $+0''\cdot40$ from 1847 to 1858, and $+0''\cdot54$ from 1862 to 1874; to these means Professor Newcomb applies corrections $-0''\cdot07$ and $+0''\cdot11$ respectively for his empirical term, thus obtaining $+0''\cdot33$ and $+0''\cdot65$ (p. 29); the corrections resulting from the actual term (calculated as in the case of the term $+0''\cdot316 \sin (g+2\varpi-3J+7^\circ)$) are $-0''\cdot22$ and $+0''\cdot03$ respectively. Hence his resulting values of the principal elliptic coefficient require correction on this account,

Ref. No.	Correction.
	″
2	+0·15
3	+0·08

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(3) $-0''{\cdot}68 \sin(g+2\varpi+3V-5E)$. This is a *Venus* term calculated by M. Radau. Its verification from the observations proved very troublesome, as there is a periodic term running through the errors with argument, Moon's longitude, and very probably arising from an erroneous tabular lunar parallax. This term gains one revolution a century upon the *Venus* term, and in *Monthly Notices*, vol. lxv. pp. 136 and 148, I have succeeded in fixing its coefficient at 0″·7. These two terms were not discovered by Professor Newcomb at the time when he discovered the *Jupiter* evection term, because about the year 1870 the two terms as nearly as possible cancel each other. On account of the two terms I have applied the following corrections :

Ref. No.	Correction for *Venus* Term.	Correction for Empirical Term.	Sum.
	″	″	″
2	+0·12	−0·08	+0·04
3	−0·11	+0·13	+0·02

The corrections are small, partly because the terms nearly cancel and partly because the length of the wave introduced in the values of the coefficients of sin g is nine years, and is therefore more thoroughly eliminated from a twelve or thirteen year discussion than a seventeen year term is.

It is worth noting that though the two terms cancel in 1870, yet in 1853 their epochs differ by 60°, and the resultant term has a coefficient of 0″·7, equal to either component term and giving rise to a negative minimum in the values of h at the epoch 1851·6 about. Consequently the values of h, k are not in such good accordance from 1847 to 1858 as they are from 1862 to 1874. This fact Professor Newcomb has evidently observed, but the true explanation, viz. the coexistence of two unsuspected terms of nearly the same period of course could not have been discovered on the evidence. He therefore suspects the earlier observations of being decidedly less accurate, and he writes (p. 29): "I consider that the results of the second series are entitled to three times the weight of those of the first."

Adding together the foregoing three corrections the totals are :

Ref. No.	Correction.
	″
2	+0·06
3	+0·10
4	+0·06

These are the quantities applied at the beginning of this paper.

In *Memoirs R.A.S.* vol. xlviii. p. 312, Mr. Nevill writes : "The probable error of a single year's result may be taken as ±0″·40." This would indicate a probable error of about ±0″·12 to the second, third, and fourth results. This is not inconsistent with a discordance of 0″·28 (decreased from 0″·32) between

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Professor Newcomb's two results. Professor Newcomb's second result and Mr. Nevill's are now in close agreement, as is only natural seeing that Mr. Nevill's sixteen years include the thirteen years of Professor Newcomb's second result. The discordance has been decreased from 0″·18 to 0″·08 by the calculations of this paper.

Note on Diurnal Variations of the Nadir and Level of the Transit Circle at the Royal Observatory, Greenwich.

(*Communicated by the Astronomer Royal.*)

In a former paper communicated to the Society in 1899 March it was shown that when the observations of the level and nadir taken at different times of the same day were compared with each other, those observations made about 6 P.M. showed discordances from those made about midday and midnight. The present paper continues these comparisons of the level and nadir near noon, 6 P.M., and midnight for the years 1897–1904. The results for these times are derived from about 100 days in each year in which one observation at least falls within the limits of each group, viz. 9^h–15^h, 15^h–21^h, and 21^h–23^h civil time. In order to carry the discussion through the whole twenty-four hours, observations made between 3^h–9^h have been compared with those obtained between 15^h–21^h on the same or the previous days. There are approximately thirty days in each year when such observations have been made. The results are given in the following table:

Diurnal Changes of Level and Nadir for the years 1897–1904.

Year.	Level.				Nadir.			
	Noon.	6 p.m.	Midnight.	6 a.m.	Noon.	6 p.m.	Midnight.	6 a.m.
	″	″	″	″	″	″	″	″
1897	+0·20	·00	+ ·13	+ ·30	+ ·11	·00	+ ·17	+ ·33
1898	+0·33	·00	+ ·23	+ ·36	+ ·16	·00	+ ·11	+ ·05
1899	+0·29	·00	+ ·18	+ ·30	+ ·17	·00	+ ·19	+ ·24
1900	+0·29	·00	+ ·23	+ ·46	+ ·14	·00	+ ·05	+ ·02
1901	+0·28	·00	+ ·26	+ ·31	+ ·17	·00	+ ·09	+ ·06
1902	+0·21	·00	+ ·20	+ ·34	+ ·20	·00	+ ·17	+ ·09
1903	+0·14	·00	+ ·22	+ ·32	+ ·04	·00	+ ·09	+ ·06
1904	+0·27	·00	+ ·29	+ ·55	+ ·06	·00	+ ·15	+ ·12
Mean	+0·25	·00	+0·22	+0·37	+0·13	·00	+0·13	+0·12

The variation of the level has a period of 24^h with its maximum about 6 A.M. and minimum about 6 P.M. The variations of the nadir are much smaller, and do not show any conclusive result except the discordance near 6 P.M.